Mathematics 8

Learn EveryWare

Unit 2
Integers

we encourage

Alberta Education

Mathematics 8

Learn EveryWare

Unit 2

Integers

Mathematics 8
Unit 2: Integers
Student Module Booklet
ISBN 978-0-7741-3132-2

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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Unit 2: Integers

Unit 2 Introduction

You may know the Rocky Mountains as a beautiful destination for skiing or hiking. But these mountains are also important in another way. They affect the weather patterns of Western Canada and likely the region where you live.

For example, weather patterns can be affected by air temperature. The set of numbers known as integers is important in measuring and recording temperatures.

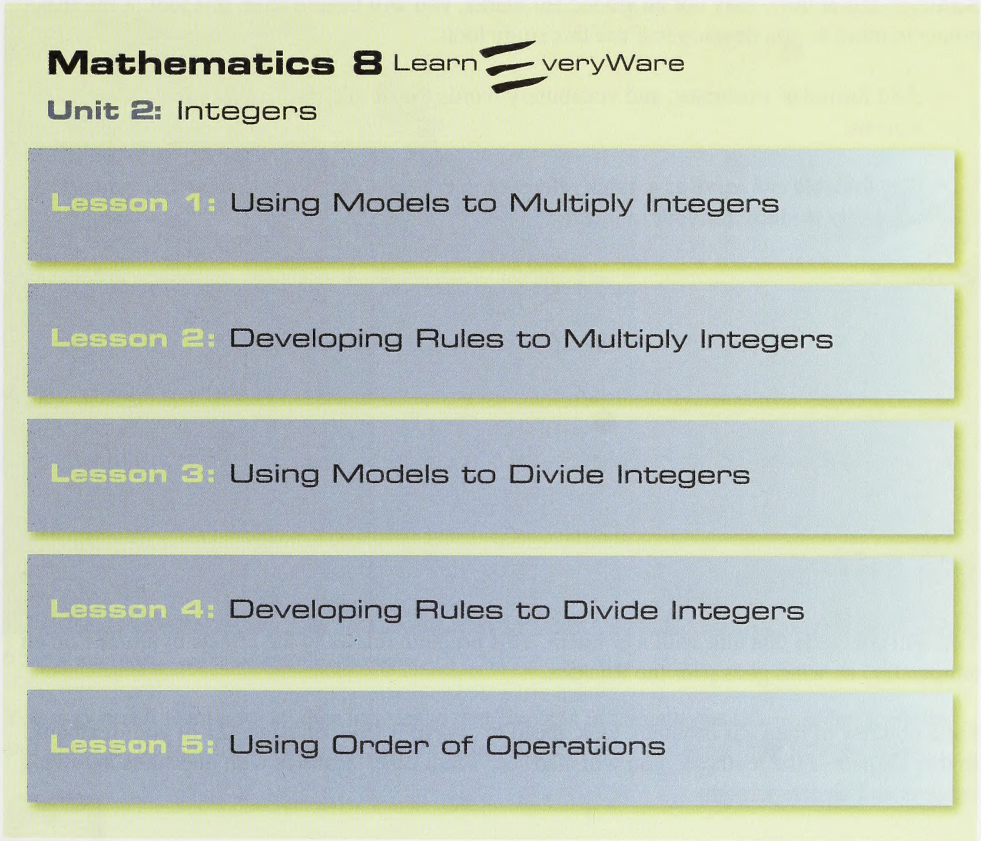
In Unit 2 you will study integers and discover how to multiply and divide with them. As you progress through this unit, you will gain the knowledge and skills needed to use integers in the explanation of weather patterns in Western Canada.

In this unit you will multiply and divide integers using models, diagrams, and symbols. You will also use a handheld calculator to complete operations with integers.

This unit will help you answer the following critical question: How can operations with integers be used to describe temperature changes in weather patterns?

When you look at the two-page spread on pages 282 and 283 of your textbook, you see a lush forest growing up to the mountains on the horizon. Rain must be plentiful on the forest side of the mountains. Do the mountains affect weather patterns? Yes, they do, according to the explanation that introduces this unit. The boxed illustration of the wind blowing over mountains helps to show why mountains have such an influence on weather patterns.

The new concepts and skills with integers will be presented in five lessons.



Keep up to your assignments as you go through this unit. Be sure to submit your assignments according to the directions provided.
You will do various types of assignments:

- posting to the discussion board
- adding samples of your work to your Math 8 folder
- completing sets of questions for each lesson
- solving a unit problem at the end of the unit
- possibly writing a unit test

Strategies for Success

In order to support your success in this unit, follow these strategies.

Strategy 1

Make a foldable study tool according to the detailed instructions on page 284 of your textbook. Although this activity may not be graded for marks, you will benefit from this tool. Keep these points in mind as you develop and use this study tool:

- Add formulas, examples, and vocabulary words to the foldable as you work through the lessons.
- The foldable can serve as a quick reference guide and will help you save time when you are ready to study for your unit test.

Strategy 2

In this unit you will be referring to pages 282 to 323 of your textbook.

- Take time to flip through these textbook pages.
- Look at illustrations, margin features, and main titles to get a sense of where you will be going.

Unit 2 Problem

You will conclude this unit with a problem. This problem relates to the effects of altitude on air temperature as wind goes over the Rockies.

For a preview of the Unit Problem, look ahead to "Wrap It Up!" on page 321 of your textbook. In this chapter of the textbook, you will also see "Math Link" features with questions involving integers and air temperature.

Unit 2: Integers

Lesson 1: Using Models to Multiply Integers

Get Focused

You may recall from your studies in science that changes in climate conditions affect icefields and glaciers. In fact, the Athabasca Glacier in Jasper National Park is melting little by little.

Read

Turn to page 286 of your textbook *MathLinks 8*. To find out more about the Athabasca Glacier, read up to "Explore the Math."

In this lesson you will model the process of multiplying two integers. You will use integer chips and pictorial representations.

This lesson will help you answer the following critical questions: How can you make sense of the operation of multiplication with integers? How can a model help?

You will need a set of 24 red and 24 blue integer chips for the lessons in Unit 2. Using integer chips will give you greater insight into how integers can be used.

You can cut out paper chips from "Integer-Chip Template" found on the Math 8 Multimedia DVD. Colour the chips red or blue before cutting them out. Another option is to use red and blue bingo chips or buttons. Whatever you decide to use, keep the chips in plastic bags or other containers for use throughout this unit. Alternately, you may use the simulation "Integer Chips" instead of chips you cut out. This simulation is also available from the Math 8 Multimedia DVD.

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 2: Lesson 1 Question Set

Explore

How far will the Athabasca Glacier recede in four years? To answer this question, you will need to multiply with integers.

To find out how you can multiply with integer chips, do "Explore the Math" on pages 286 and 287 of your textbook. Work with a partner, if possible. Your partner may be a classmate, friend, or family member.

Observe how a red and a blue integer chip are related to each other.

Self-Check

SC 1. Complete questions 1, 2, 3, and 4 of "Explore the Math."

Compare your answers in the Appendix.

SC 2. Complete question 5 of "Explore the Math" on page 287 of your textbook.

My Guide

Q: How is the first factor different from the ones in previous questions?

A: The first factor is a negative integer rather than a positive number.

SC 3. Complete questions 6.a), 6.b), 7.a), and 7.c) of "Explore the Math" on page 287 of your textbook.

Compare your answers in the Appendix.

Try This

TT 1. Complete question 8 of "Explore the Math" on page 287 of your textbook.

Discuss and Share

Post your response to TT 1 on the discussion board. Then respond to at least two other postings.

Connect

Let's now build on what you've found out in the Explore.

Read

Read "Example 1: Multiply Using Integer Chips" on pages 288 and 289 of your textbook to see how integer chips can be used to find products. Watch to see when you need to use both red and blue integer chips.

Self-Check

SC 4. Do the "Show You Know" questions in the middle of page 289 of your textbook.

Compare your answers in the Appendix.

Read

Read "Example 2: Apply Integer Multiplication" on page 289 of your textbook to see how you can solve problems with integer multiplications.

My Guide

Q: Why is the change in time represented by a positive integer?

A: The time is represented by a positive integer because it increases.

Q: Why is the temperature change represented by a negative integer?

A: In each hour, the temperature drops or decreases.

Q: Why is multiplication used to find the total change?

A: For 5 h, the temperature changes by −3°C in each hour. The total change can be represented by the repeated addition (−3) + (−3) + (−3) + (−3) + (−3). This expression equals (+5) × (−3).

Self-Check

SC 5. Do the "Show You Know" question at the bottom of page 289 of your textbook.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on page 290 of your textbook to see how you can solve problems with integer multiplications.

SC 6. When do you place zero pairs of integer chips to model the multiplication of integers?

Compare your answers in the Appendix.

Try This

TT 2. Do question 3 from "Communicate the Ideas" on page 290 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 7. Do "Practise" questions 6, 8, 9.a), 10.a), 11.a), and 12.a) on pages 291 and 292 of your textbook.

Compare your answers in the Appendix.

Self-Check

SC 8. Do "Apply" questions 15 and 17 on page 292 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Are you still struggling with the concepts of this lesson? Then you may want some extra practice.

Turn to pages 291 and 292 of your textbook. For extra practice, you may complete more questions from "Check Your Understanding." Then check your work using the answers given at the back of your textbook.

Assignment

Go to the Unit 2 Assignment Booklet, and complete "Unit 2: Lesson 1 Question Set."

Save your answers to this question set. You are to submit your answers according to instructions in one of the lessons that follow.

Going Beyond

Look at the following square of numbers. Because it has the same number of rows as columns, it's called a *square*. Notice that the rows, columns, and diagonals all have the same sum. That characteristic makes this a magic square.

−2	3	2
5	1	−3
0	−1	4

Complete "Extend" question 19 on page 292 of your textbook to find out what happens when you use integer multiplication to change a magic square.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you used models to represent the process of multiplying two integers. You based your models on arrangements of integer chips.

You found that you could make sense of multiplication of any integer by a positive integer. You made sense of this by thinking of multiplication as repeated addition. The number of addends showed the number of groups of integer chips to insert. The integer value of the addend lets you know the number and colour of the integer chips in each group being added.

You found that you could also make sense of multiplication of an integer by a negative integer. You used a similar way to model the product. But here you thought of multiplication by a negative integer as repeated removal of groups of integer chips. But you had to insert enough zero groups so that you would have enough integer chips to take away. The value of the remaining integer chips represented the product.

Unit 2: Integers

Lesson 2: Developing Rules to Multiply Integers

Get Focused

Changing weather conditions trigger birds to start migration. During the autumn, you've probably seen geese flying high overhead as they migrate south in their well-known V-shaped formation. You may not be familiar with the sooty shearwater, but the sooty shearwater is also a migratory bird. What sets this bird apart from other birds is that the length of its migratory path is longer than any other.

Read

Turn to page 293 in your textbook *MathLinks 8*. To find out more about the sooty shearwater, read up to "Explore the Math."

Integers are useful in describing the diving behaviour of sooty shearwater and also one aspect of weather—air temperature.

In this lesson you will model the process of multiplying two integers using a number line. You will also develop sign rules for multiplying integers.

This lesson will help you answer the following critical question: Is there a rule that tells what the sign of the product of integers should be?

Again, you will need the 24 red and 24 blue integer chips you made in the previous lesson. If you still need to make them, cut them out from the "Integer-Chip Template."

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 2: Lesson 2 Question Set

Explore

What are the deepest dives of a sooty shearwater? You can calculate the answer to this question by multiplying with integers.

To find out how you can use a number line to multiply integers, complete "Explore the Math" on pages 293 and 294 of your textbook. You will try to develop a rule that will tell whether a product will be a positive integer or a negative integer. If you think you know such a rule, write it down on paper so you can test it and see if it is true. Work with a partner, if possible. Your partner may be a classmate, friend, or family member.

Self-Check

SC 1. Complete questions 1, 2, and 3 of "Explore the Math" on pages 293 and 294 of your textbook.

Compare your answers in the Appendix.

SC 2. Complete question 4 of "Explore the Math" on page 294 of your textbook.

My Guide

Q: How do the multiplication statements in the first column differ from those in the second column?

A: The multiplication statements in the second column have the factors switched around.

Compare your answers in the Appendix.

Try This

TT 1. Complete questions 5.a) and 6.a) of "Explore the Math" on page 294 of your textbook.

Discuss and Share

Post your response to TT 1 on the discussion board. Then respond to at least two other postings.

Connect

Read

Read "Example 1: Multiply Integers" on pages 294 and 295 of your textbook to see how sign rules can help you find products of integers really quickly. It would be a great idea to write these rules in your own words with examples that help you understand the rules.

Self-Check

SC 3. Do the "Show You Know" questions in the middle of page 295 of your textbook.

Compare your answers in the Appendix.

Read

The sign rule for multiplication of integers helps you to solve math problems that involve money. Find out how by reading "Example 2: Apply Integer Multiplication" on page 295 of your textbook.

Self-Check

SC 4. Do the "Show You Know" question at the bottom of page 295 of your textbook.

Compare your answers in the Appendix.

Read

For a summary of the new concepts of this lesson, read "Key Ideas" on page 296 of your textbook. You will see a summary of how to multiply integers using a number line or a sign rule.

Self-Check

SC 5. Jacob entered the following multiplication statement.

(__ 11) × (__ 7) = −77

Jacob did not enter the signs for the integer factors.

Indicate what the integer values of the factors could be.

Compare your answers in the Appendix.

Try This

TT 2. Do questions 1 and 3 from "Communicate the Ideas" on page 296 of your textbook. You may use the simulation "Integer Arrows" on the Math 8 Multimedia DVD to help you.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 6. Do "Check Your Understanding" questions 4, 5, 7, 9.a), 9.d), 10.a), 10.d), 13, 19, and 23 on pages 297 and 298 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Are you still struggling with the concepts of this lesson? Then you may want some extra practice.

Turn to pages 297 and 298 in your textbook. For extra practice, you may complete additional questions from "Practise" and "Apply." Then check your work using the answers given at the back of your textbook.

Assignment

Go to the Unit 2 Assignment Booklet, and complete "Unit 2: Lesson 2 Question Set."

Submit your answers to the Lessons 1 and 2 question sets to your teacher for marking.

Going Beyond

In regular magic squares, the rows, columns, and diagonals all have the same sum. Look at the following arrangement of numbers. The rows, columns, and diagonals all have the same product, rather than the same sum. So this table of numbers is called a *magic multiplication square*.

−12	−9	−2
−1	−6	−36
−18	−4	−3

Complete the following question to find out what happens when you use integer operations to change a magic multiplication square.

Turn to page 299 of your textbook and complete question 28.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you modelled the process of multiplying two integers using a number line and discovered a sign rule for multiplying integers.

The sign rule tells what the sign of the product should be when multiplying with two integer factors. The rule states that if the sign of the factors are the same, then the sign of the product is positive. If the sign of the factors are different, then the sign of the product is negative.

Unit 2: Integers

Lesson 3: Using Models to Divide Integers

Get Focused

Grizzly bears spend the winter months in their dens. When the temperature drops due to autumn weather patterns, the bears become sleepy. Then, usually during a snow storm, they enter their dens a final time before their long winter sleep of several months. Only when the warm temperatures of spring arrive do the bears come out of their winter sleep.

Read

To learn about the changes in a bear's body mass during a bear's winter sleep, turn to page 300 in your textbook *MathLinks 8*. Read up to "Explore the Math."

Integers and integer division are useful in describing both the rate at which a bear's body mass changes and the rate at which temperature changes.

In this lesson you will model the process of integer division by using integer chips.

This lesson will help you answer the following critical question: How can a model provide meaning to the division of integers?

Again, you will need the 24 red and the 24 blue integer chips you made for this unit. If you still need to make them, cut them out from the "Integer-Chip Template." Alternately, you may want to use the simulation "Integer Chips" instead of chips you cut out.

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 2: Lesson 3 Question Set

Explore

How much body mass does a grizzly bear lose each month of its winter sleep? You can calculate the answer to this question by dividing with integers.

To find out how you can use integer chips to model the process of division with integers, complete "Explore the Math" on pages 300 and 301 of your text. Work with a partner, if possible. Your partner may be a classmate, friend, or family member.

My Guide

See examples of a dividend, a divisor, and a quotient in the division statements. Go to the Math 8 Multimedia DVD, and look at "Dividend," "Divisor," and "Quotient."

Self-Check

SC 1. Complete questions 1, 2, and 3 of "Explore the Math" on pages 300 and 301 of your textbook.

Compare your answers in the Appendix.

My Guide

In general, a division statement such as (+10) ÷ (+2) = ? can be translated in two ways:

- Translation 1: How many groups of 2 are there in 10 integer chips?
- Translation 2: How many chips are in each group when 10 integer chips are put into 2 groups?

When the divisor is a negative integer, only one of these translations helps you to model the division statement with integer chips.

Q: Which translation does not help you model the division statement (−15) ÷ (−5) with integer chips? Explain.

A: Translation 1 does not help. It would translate the division statement as this: How many are in each group when 15 blue chips are divided into –5 groups? Separating chips into a negative number of groups is not very meaningful.

Q: Translate the division statement into a form that you can model with integer chips.

A: (−15) ÷ (−5) = ? should be translated into this form: How many groups of integer chips will you form when you make as many groups of 5 blue integer chips as you can from the 15 blue integer chips you start with?

SC 2. Complete question 4 of "Explore the Math" on page 301 of your textbook.

Compare your answers in the Appendix.

My Guide

Q: How do you know whether to use blue or red integer chips to model a division statement?

A: You look at the sign of the dividend. If it is negative, you use blue integer chips; if it is positive, you use red ones. Note that here in these lessons, blue is negative and red is positive. However, if you were working by yourself, you could decide on any colours for negative and positive chips, as long as you were consistent. It is best to stick with the red and blue colour here, so everyone is working with the same colours and there is less confusion.

Q: Suppose you tried to find the quotient for (−8) ÷ (+2) = ? by seeing how many groups of red integer tiles you can make. Why would this NOT work?

A: The dividend is negative, so you only have blue integer tiles to put into groups. You cannot form any groups of red tiles.

Q: What should you be doing with the 8 blue tiles to find the quotient for $(-8) \div (+2) = ?$

A: You should see what the make-up of a group is when you separate the 8 blue integer tiles into 2 groups.

SC 3. Complete question 5 of "Explore the Math" on page 301 of your textbook.

Compare your answers in the Appendix.

My Guide

Q: Why do you think have you not been asked to model division with a positive dividend and a negative divisor? For example: $(+6) \div (-2)$

A: A negative divisor could indicate a negative number of groups to separate integer chips into. That does not make much sense. Also, you cannot look for groups having any number of blue tiles among the red tiles. There just are no blue tiles to put into groups.

Try This

TT 1. Complete question 6 of "Explore the Math" on page 301 of your textbook. Think of the different combinations of positive and negative integer dividends and divisors a division statement can have. Your answer may consist of a description of solutions to sample division statements.

Discuss and Share

Post your response to TT 1 on the discussion board. Then respond to at least two other postings.

Connect

Read

The next reading shows there is more than one strategy you can use to model division. Read "Example 1: Divide Using Integer Chips" on pages 301 and 302 of your textbook to see the ways integer chips can be used for division.

Self-Check

SC 4. Do questions a) and b) of "Show You Know" in the middle of page 302 of your textbook.

Compare your answers in the Appendix.

Read

One night in Wetaskiwin, a drop of 2°C every hour was recorded. Read "Example 2: Apply Integer Division" on page 302 of your textbook to see how you can use integer division to find out more about the cooling that occurred that night.

Self-Check

SC 5. Do the "Show You Know" question at the top of page 303 of your textbook.

Compare your answers in the Appendix.

Read

For a summary of how an integer-division statement can be modelled and solved using integer chips, read "Key Ideas" on page 303 of your textbook.

Self-Check

SC 6. Suppose the integer chips in the set of diagrams in "Key Ideas" were red instead of blue. What two integer statements would be modelled?

Compare your answers in the Appendix.

Try This

TT 2. Do questions 1.a), 1.b), and 1.d) from "Communicate the Ideas" on page 303 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 7. Do "Practise" questions 3, 4, and 6 on page 304 of your textbook.

Compare your answers in the Appendix.

SC 8. Do "Apply" questions 11 and 12 on page 305 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Do you still find multiplication of integers using integer chips a bit tricky? Then extra practice will help.

Turn to pages 304 and 305 in your textbook. For extra practice, you may complete additional questions from "Practise" and "Apply." Then check your work using the answers given at the back of your textbook.

Assignment

Go to the Unit 2 Assignment Booklet, and complete "Unit 2: Lesson 3 Question Set."

Save your answers to this question set. You are to submit your answers according to instructions you will see in a later lesson.

Going Beyond

You have seen that integer division allows you to find rates of change. For example, you found a rate of temperature change and a rate of change in body mass. Knowing a rate of change may help you make predictions. See how to predict a temperature in a winter's night using integer division and other integer operations.

Complete the following question to predict a night-time temperature. Turn to page 305 of your textbook and do "Extend" question 16.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you used integer chips to model the process of dividing one integer by another. You also solved problems with integer division. You found modelling with integer chips provided meaning to the operation of division with integers. Modelling involves making equal groups or separating into a certain number of groups. The arrangement of the integer chips shows the value of the quotient.

Lesson 4: Developing Rules to Divide Integers

Get Focused

Temperature changes you experience are due to the present-day weather and climatic patterns in Canada. But at one time long ago, the climate on the Prairies was tropical. The water of an inland sea covered the land, and warm-air temperatures helped the water to evaporate. Potash was one of the minerals left behind when the sea water evaporated. Potash is an important ingredient in many plant fertilizers.

Nowadays, miners have to travel a great distance to reach potash deposits now found far under ground.

Read

Turn to page 306 in your textbook *MathLinks 8*, and read up to "Explore the Math" to find out more about the miners' journey.

Integers and integer division are useful in describing temperatures as well as the up and down movement of miners in their elevator cage.

In this lesson you will model the process of dividing one integer by another using a number line. From your division with a number line, you will develop sign rules for division of integers. You will also start using a calculator to help with your division.

This lesson will help you answer the following critical question: Is there a rule that tells what the sign should be when dividing integers?

You will continue to use the 24 red and the 24 blue integer chips you made earlier for this unit. If you need to, you can cut out paper "chips" from the "Integer-Chip Template." Throughout this lesson, you may use the simulation "Integer Arrows."

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 2: Lesson 4 Question Set

Explore

How long do potash miners ride their cage to the potash deposits that are almost one kilometre underground? You can calculate the answer to this question by dividing with integers.

To find out how you can use a number line to model the process of division, do "Explore the Math" on page 306 and 307 of your text. Work with a partner, if possible. Your partner may be a classmate, friend, or family member.

Self-Check

SC 1. Complete questions 1, 2, 3, 4, 5, and 6 of "Explore the Math" on pages 306 and 307 of your textbook.

Compare your answers in the Appendix.

My Guide

Q: Why do you think you have not been asked to use a number line for division with a positive dividend and a negative divisor? Example: $-6 \div +2$

A: A negative divisor could indicate a negative number of sections to separate an arrow into. That does not make much sense. Neither can you describe any blue arrow sections of a red arrow. There just are no blue sections of a red arrow.

Try This

TT 1. Complete question 7 of "Reflect on Your Findings" on page 307 of your textbook.

Discuss and Share

Post your response to TT 1 on the discussion board. Then respond to at least two other postings.

Connect

Read

See how division with a number line leads you to a handy sign rule. Read "Example 1: Divide Integers" on page 308 of your textbook.

My Guide

"Example 1: Divide Integers" shows the  key used to enter a negative integer.

On other calculators, the negative key may look like this: 

Self-Check

SC 2. Do the "Show You Know" questions at the bottom of page 308 of your textbook.

Compare your answers in the Appendix.

Read

Imagine going to a restaurant with some friends. One person may pay the bill, but you and your friends agree that you will share the cost evenly. That can lead to an interesting division problem. Read "Example 2: Apply Integer Division" on page 309 of your textbook to see how you can solve such a problem using integer division.

Self-Check

SC 3. Do the question from "Show You Know" near the middle of page 309 of your textbook.

Compare your answers in the Appendix.

Read

Think of two main ideas of this lesson. See if you can find the main ideas you thought of in the next reading. Read "Key Ideas" on page 309 of your textbook for a summary of the lesson.

Self-Check

SC 4. Suppose the blue arrow in the diagram in "Key Ideas" was moved and changed to a red arrow pointing to the right. What two integer statements would be modelled then?

Compare your answers in the Appendix.

Try This

TT 2. Do questions 1 and 2 from "Communicate the Ideas" on pages 309 and 310 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 5. Do "Practise" questions 5, 6, 8.a), 8.c), 9.a), 9.c), 10.a), and 10.c) on page 310 of your textbook.

SC 6. Do "Apply" questions 12, 16, and 19 on pages 310 and 311 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Turn to pages 310 and 311 in your textbook. For extra practice, you may complete additional questions from "Practise" and "Apply." Then check your work using the answers given at the back of your textbook.

Assignment

Go to the Unit 2 Assignment Booklet, and complete "Unit 2: Lesson 4 Question Set."

Submit your answers to the Lessons 3 and 4 question sets to your teacher for marking.

Going Beyond

Do you enjoy riddles with numbers? Suppose you are looking for two whole numbers that add up to 12. When you divide the larger number by the smaller one, the quotient is 2. What are the two whole numbers?

After some guess and check, you may soon find that the whole numbers are 8 and 4.

Riddles involving integers can give you even more challenge. Turn to page 311 of your textbook for an integer riddle, and try "Extend" question 22.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you modelled the process of dividing one integer by another using a number line. As well, you developed sign rules for the division of integers and used your calculator to find quotients.

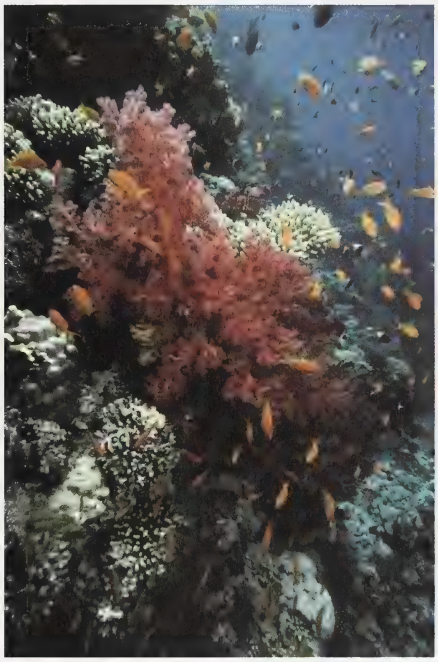
The sign rule for division is as follows: When dividing with two integers having the same sign—whether the integers are either both positive or both negative—the quotient will be positive. If you are dividing with two integers having opposite signs, the quotient will be negative.

Unit 2: Integers

Lesson 5: Using Order of Operations

Get Focused

The temperatures that you experience can vary every day. The temperature that sea life experiences is also changeable. Scientists have gone to great depths to study temperature and other conditions below the ocean surface. You know integers are used to describe temperatures. Integers are also used to describe heights and depths measured from the surface of the ground or ocean.



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Read

Turn to page 312 in your textbook *MathLinks 8*, and read up to "Explore the Math" to find out more about a famous vehicle used to travel deep below the waves.

Integers and integer operations are useful in describing temperatures as well as depths of the ocean.

In Lesson 5 you will solve problems involving integers, taking into consideration the order of operations.

This lesson will help you answer the following critical question: When an expression involves several operations with integers, in what order should they be done?

You may use a calculator to help you with calculations.

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 2: Lesson 5 Question Set

Explore

How far does a submersible reach after diving at known speeds? You can calculate the answer to this question by using an expression having more than one operation in it. But to use this expression, you need to do the operations in a sensible order.

Why do you have to pay attention to the proper order of integer operations? To find out, do "Explore the Math" starting on page 312 of your textbook and finishing just before "Example 1: Use the Order of Operations" on page 313. Find out if you get the same answer as Laura, Abeni, or Rob, or an entirely different answer. Work with a partner, if possible. Your partner may be a classmate, friend, or family member.

Self-Check

SC 1. Complete questions 1, 2, 3, and 4 of "Explore the Math" on page 313 of your textbook.

Compare your answers in the Appendix.

My Guide

Q: What stage of the descent does the expression $6 \times (-15)$ represent?

A: The expression describes the first 6 minutes when the submarine went down at a rate of 15 m every minute.

Q: What stage of the descent does the expression $20 \times (-25)$ represent?

A: This stage is the final 20 minutes when the submarine went down at a rate of 25 m every minute.

Q: Once you know how far the submarine went down in each of the two stages, how do you determine the depth of the submarine at the end of the descent?

A: Add the distances travelled in each stage to find the depth of the submarine at the end of the descent.

Try This

TT 1. Complete question 5 of "Reflect on Your Findings" on page 313 of your textbook.

Discuss and Share

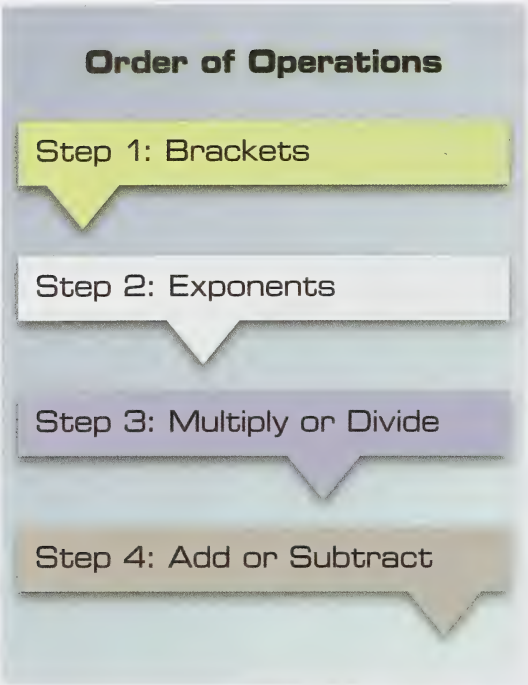
Post your responses to TT 1 on the discussion board. Then respond to at least two other postings.

Connect

Read

Go to "Example 1: Use the Order of Operations" on page 313 of your textbook to see the usual order in which you do operations.

See how the order of operations in the example fits the steps represented in the diagram.



Step 2 of the diagram mentions exponents—for example the 2 in the area formula $A = s^2$. If a mathematical expression has no exponents, you simply skip step 2.

Self-Check

SC 2. Do the "Show You Know" questions at the bottom of page 313 of your textbook.

Compare your answers in the Appendix.

Watch and Listen

Use the order of operations in the interactive game of "Order of Operations—Use It" on the Math 8 Multimedia DVD. Your score will depend on both answering the questions correctly and answering the questions quickly.

Read

Read "Example 2: Apply Integer Operations" on page 314 of your textbook to see how you can find the mean temperature using integer operations. Note that mean temperature does not refer to cruelly cold or unpleasant temperatures. Think of the mean temperature as an average of temperatures.

Self-Check

SC 3. Do the "Show You Know" question near the middle of page 314 of your textbook.

Compare your answers in the Appendix.

My Guide

Do you remember the order of operations for whole numbers and decimals? The order of operations for integers is just the same.

Read

Go to "Key Ideas" on page 314 of your textbook to read a summary of integer division. Make note of this for study purposes.

Self-Check

SC 4. When are square brackets used in an integer expression?

Compare your answers in the Appendix.

Try This

TT 2. Do questions 1 and 2 from "Communicate the Ideas" on pages 314 and 315 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 5. Do "Practise" questions 4.a), 4.b), 5.a), 5.b), 6.b), 6.c), and 7.c) on page 315 of your textbook.

Compare your answers in the Appendix.

SC 6. Do "Apply" questions 8, 11, 16.a), 17, and 20 on pages 315 and 316 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Turn to pages 315 and 316 in your textbook. For extra practice, you may complete additional questions from "Practise" and "Apply." Then check your work using the answers given at the back of your textbook.

Assignment

Go to the Unit 2 Assignment Booklet, and complete "Unit 2: Lesson 5 Question Set."

Submit your answers to the Lesson 5 question set to your teacher for marking.

Going Beyond

Riddles involving integers can be fun. In the next question you will be given an integer expression and its value. However, the operation signs are not shown and are left for you to work out.

Turn to page 317 of your textbook and do "Extend" question 21.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you solved problems involving integers and applied proper order of operations.

When an expression involves several operations with integers, the operations should be carried out in this order:

- First, complete the operations in brackets.
- Second, complete the multiplication and division in the order you see them as you read the expression from left to right.
- Third, complete the addition and subtraction in the order you see them as you read the expression from left to right.

Unit 2: Integers

Unit 2 Summary

Getting Started

The Rocky Mountains affect the weather patterns of Western Canada.

Eastward-flowing air rises in altitude as it approaches the mountains and passes over the mountain tops. The air then moves to lower altitudes again as it goes beyond the mountain tops and down the eastern slopes of the Rocky Mountains. The changes in altitude cause changes in the temperature—and also the humidity—of the air.

The Unit Problem concludes with "Wrap It Up!" on page 321 of your textbook. You use integers and integer operations to describe and predict the impact of altitude on the temperature of air flowing over the Rocky Mountains.

Read

For a sense of how integers will be used in the Unit Problem, read "Math Link" on page 285 of your textbook. Much of this "Math Link" is based on knowledge you may have had about integers before starting this unit.

My Guide

Q: Why are the disks in the diagram different colours?

A: The 3 disks in the first row represent the negative integer −3. The 8 disks in the second row represent the positive integer +8. Blue disks are used for negative integers and red ones for positive integers in this example.

Q: How can the arrangement of the disks be simplified to show the sum of +8 and −3?

A: A red disk can be paired with a blue disk to make a zero pair. You can make three zero pairs this way. The three zero pairs you make can be removed. The remaining red disks represent the sum.

Q: In questions 1 and 2 you used addition and subtraction. What other operation is needed to complete question 3?

A: In question 3 you need to divide by 4 to get the temperature change per hour. That is, the temperature change per hour can be represented the expression (−20) ÷ 4.

Q: How do you use model division of −20 by 4 using 20 blue disks?

A: Separate the 20 blue disks into 4 groups having an equal number of disks. The number of disks in each of the 4 groups represents the outcome of the division.

Self-Check

SC 1. Complete questions 1, 2, and 3 of the "Math Link" on page 285 of your textbook. If possible, work with a partner. Your partner may be a classmate, friend, or family member.

Compare your answers in the Appendix.

The Self-Check questions you just completed helped you prepare for the Unit Problem. Your project consists of responding to the questions that follow.

Project

Air cools as it travels up a mountain side, so you may expect that air at higher altitudes would be cooler than at ground level—even in the absence of nearby mountains. In fact, observations made by weather balloons show that the temperature of air generally does decrease with altitude—at least, for the first 10 km off the ground.

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Try This

TT 1. The "Math Link" on page 299 shows the relationship between air temperature and altitude. Answer questions a) and b) to get a better understanding of this relationship.

Place a copy of your answers in your Math 8 course folder.

At cruising height, passenger aircraft fly much higher than even the highest peaks of the Rocky Mountains. Did you know that at their cruising height, these aircraft fly through air that is as cold as the Arctic in winter? The difference in air temperature between the air at the flying level of an airplane and normal ground level can be used to calculate the height of the airplane. In the next question, you will be given the information you need to make such a calculation.

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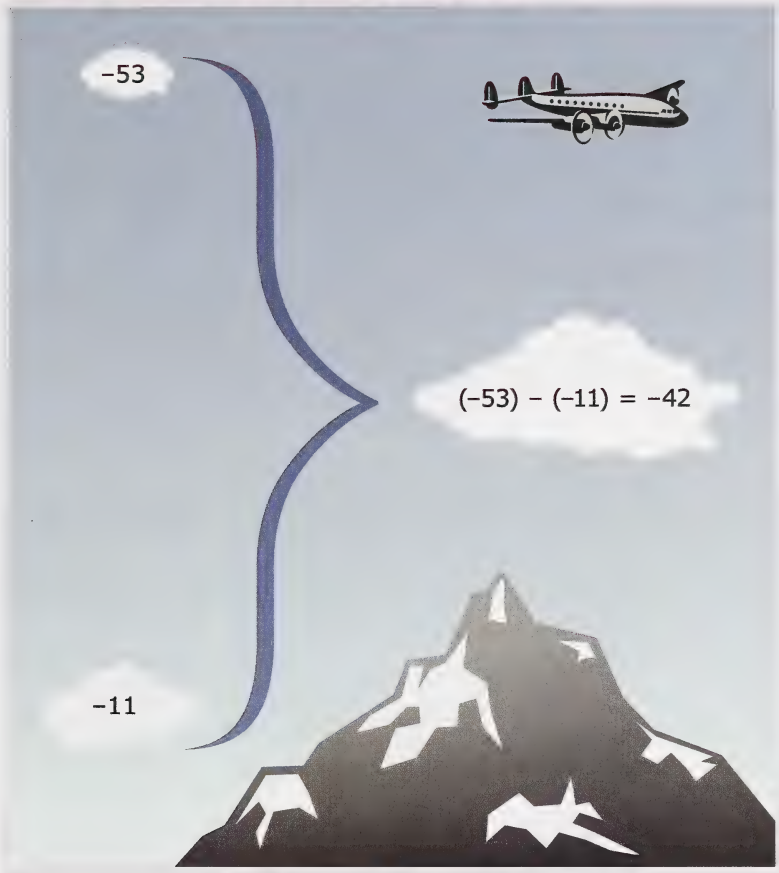
My Guide



Try This

TT 2. Complete "Math Link" questions a) and b) on page 311 of your textbook to discover how temperature difference can be used to find the flying height of an airplane.

Place a copy of your answers in your Math 8 course folder.

So how does air temperature change as it goes up and down the mountains? Knowing that will help explain why the weather in Calgary differs from the weather in Vancouver.

Turn to "Wrap It Up!" on page 321 of your textbook and read the description of air moving over the Rocky Mountains. This description comes just before the questions.

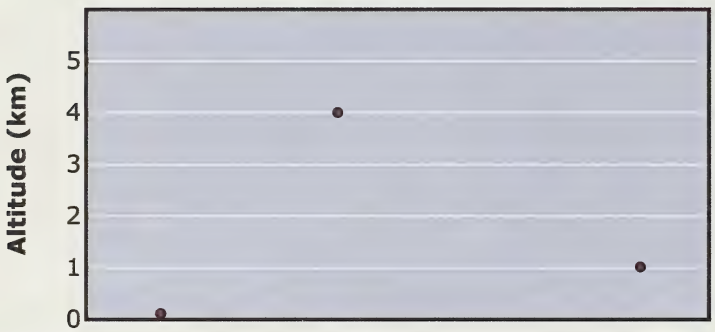
Try This

TT 3. Do questions a), b), c), and d) of "Wrap It Up!" on page 321 of your textbook.

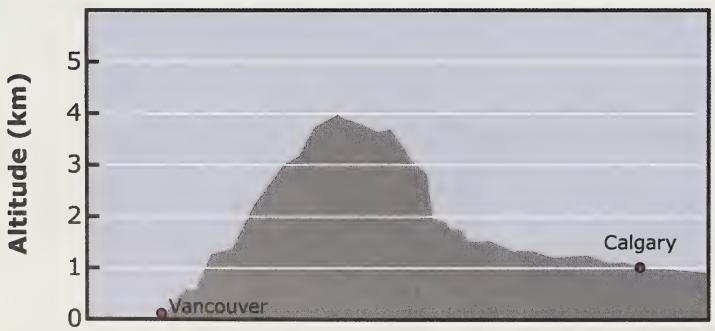
My Guide

You may find the following steps helpful.

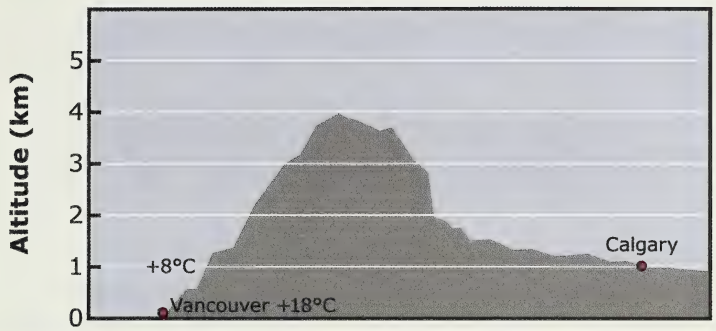
Step 1: Draw a grid first before sketching the mountains.



Step 2: Sketch the outline of the mountains on the grid like that shown in the textbook so that the top of the outline goes through a point on the 4 km line. Label Vancouver and Calgary on your outline.



Step 3: Place the temperature values up and down the mountain surface near or on the appropriate horizontal lines of the grid. The two temperature values are shown as examples.




Place a copy of your answers in your Math 8 course folder.

Unit Summary

In this unit you multiplied and divided integers using models, diagrams, and symbols. You also used integers in problem solving and used a handheld calculator to help you complete operations with integers.

This unit helped you use integer operations to describe temperature changes in weather patterns. You saw that negative integers can be used to represent temperatures below the freezing point of water and positive integers for temperatures above this point. Temperature changes may be found by subtraction of integers, rates of temperature changes by division of integers, and final temperatures by multiplication of integers.

Unit Review

In order to reinforce the concepts and skills of this unit, complete some questions that make you think back to each of the lessons of this unit.

Self-Check

SC 3. Turn to "Chapter 8 Review" on pages 318 and 319. Complete questions 1, 2, and 3 and answer at least two questions from each of the five lesson components. You may do additional questions for practice.

Compare your answers in the Appendix.

Are You Ready?

You've done a lot of work to reach this point in the unit. But are you ready to strut your stuff—that is, show your mastery of the new concepts and skills? Are you really ready to take the challenge of a unit test?

You may feel you are ready but you want to do a practice test before doing one for marks. Well, here's your chance. Complete the following Self-Check question.

Self-Check

SC 4. Turn to pages 320 and 321 and complete "Practice Test."

Compare your answers in the Appendix.

Assignment

Check with your teacher about a unit test.

Appendix

Lesson 1

SC 1.

1. a. 

There are 3 groups of 2 red integer chips. In total, there are 6 red integer chips.

b. So $(+3) \times (+2) = +6$

2. a. $(+4) \times (+3) = (+3) + (+3) + (+3) + (+3)$

b. 

c. $(+4) \times (+3) = +12$

3. a. $(+3) \times (-5) = (-5) + (-5) + (-5)$

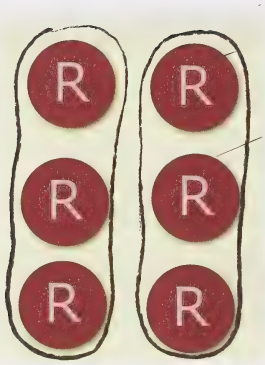
b. 

c. $(+3) \times (-5) = -15$

4. a. 

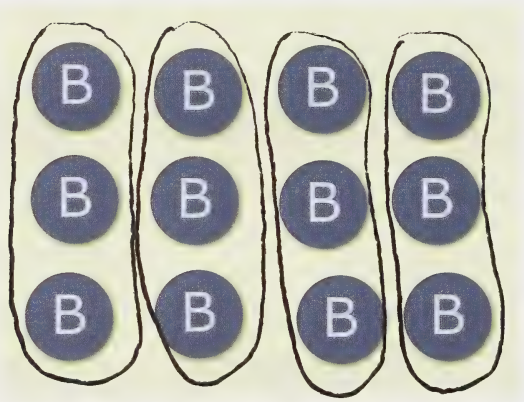
$(+3) \times (+5) = +15$

b.



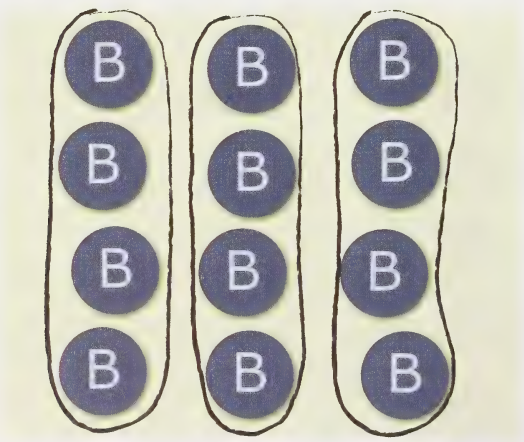
$(+2) \times (+3) = +6$

c.



$(+4) \times (-3) = -12$

d.



$(+3) \times (-4) = -12$

SC 2. Answers will vary.

5. The first factor is negative. In the previous multiplications, you could use the first factor to tell you how many groups you had to make. A negative number of groups cannot be interpreted the same way.

   If the first factor is −2, for example, then this can be interpreted as having to take 2 groups away. Similarly, a first factor of −3 could mean taking away 3 groups.

   Somehow, a different method must be used since you need to have some groups of integer tiles in place in order to take them away.

SC 3.

6. a. Modelling $(-2) \times (+3) =$

   Use 2×3, which equals 6, zero pairs of integer chips.

   Then take away 2 groups of 3 red integer chips.

   This leaves 6 blue integer chips.

   Modelling $(-3) \times (-4) =$

   Use 3×3, which equals 12, zero pairs of integer chips.

   Then take away 3 groups of 4 blue integer chips.

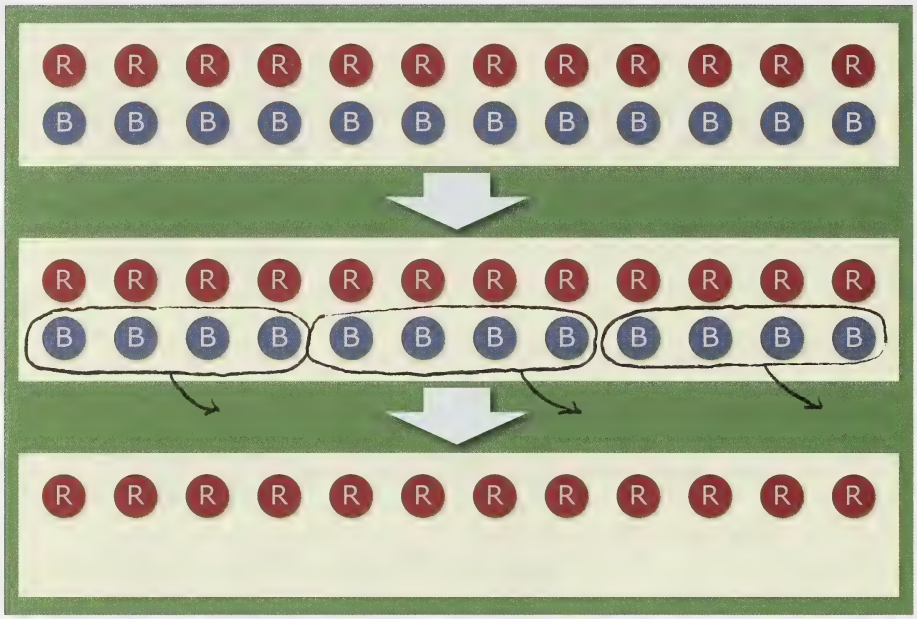
   This leaves 12 red integer chips.

   b. $(-2) \times (+3) = -6$

   $(-3) \times (-4) = +12$

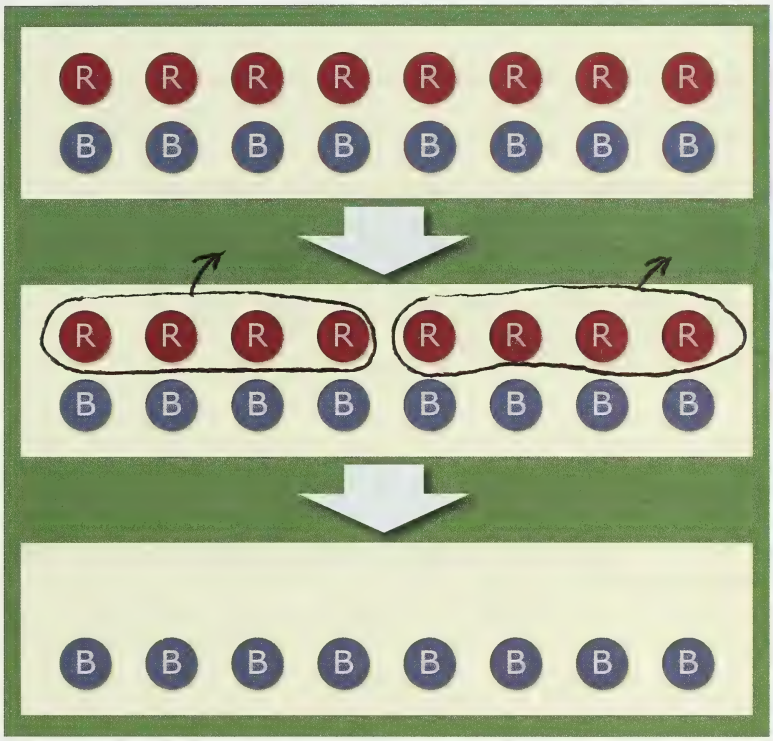
7.

a.



$(-4) \times (-3) = +12$

7. c.



$(-2) \times (+4) = -8$

SC 4.

a. By placing 4 groups of 2 red integer pairs, you have 8 red integer chips.

$(+4) \times (+2) = +8$

b. By placing 5 groups of 2 blue integer pairs, you have 10 blue integer chips.

$(+5) \times (-2) = -10$

c. After placing 8 zero pairs and then taking away 4 groups of 2 red integer chips, you are left with 8 blue integer chips.

$(-4) \times (+2) = -8$

d. After placing 6 zero pairs and then taking away 6 groups each consisting of 1 blue integer chip, you are left with 6 red integer chips.

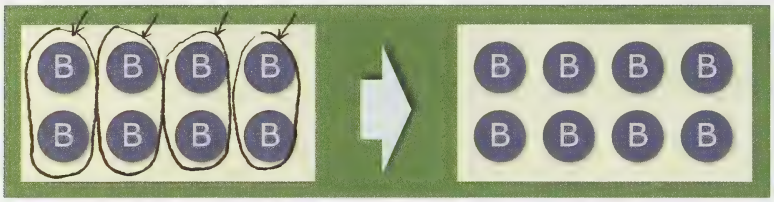
$(-6) \times (-1) = +6$

SC 5. Represent the change in time by +4.

Represent the change in temperature each hour by −2.

Then the total change in temperature should be represented by the product (+4) × (−2).

Find the product of (+4) × (−2) by using integer chips.



The product is −8.

The total change in temperature was a decrease of −8°C.

SC 6. When the first factor of the multiplication statement is a negative integer, you place zero pairs to model the multiplication.

SC 7.

6. a. (+3) × (+7)

   b. (+4) × (−4)

8. a. (+7) × (+2) = (+2) + (+2) + (+2) + (+2) + (+2) + (+2) + (+2)

   b. (+4) × (−9) = (−9) + (−9) + (−9) + (−9)

9. a. Placed in position are 2 groups of 4 red integer tiles. The result is 8 red tiles.

   This process models the multiplication statement (+2) × (+4) = (+8).

10. a. There are 7 groups each having 2 red integer tiles. In all, this results in 14 red tiles.

    This process models the multiplication statement (+7) × (+2) = (+14).

11.

a. There are 6 zero pairs placed. Then 3 groups are removed. Each removed group contains 2 blue integer chips.

The removal of 3 groups indicates the first factor of the multiplication statement is (−3). The second factor is (−2) because each removed group contains 2 blue integer chips.

This process results in 6 red integer chips remaining.

So the multiplication statement is $(-3) \times (-2) = (+6)$.

12.

a. There are 7 zero pairs placed. Then 1 group is removed. The removed group contains 7 red integer chips.

The removal of 1 group indicates the first factor of the multiplication statement is (−1). The second factor is (+7) because each removed group contains 7 red integer chips.

This process results in 7 blue integer chips remaining.

So the multiplication statement is $(-1) \times (+7) = (-7)$.

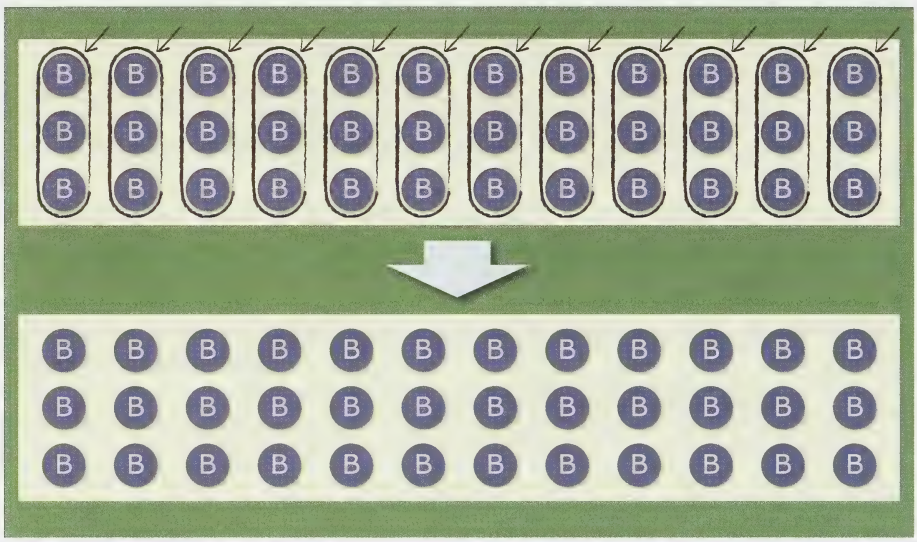
SC 8.

15. Represent the change in time by +12.

    Represent the change in altitude each second by −3.

    Then the total change in altitude should be represented by the product (+12) × (−3).

    Find the product of (+12) × (−3) by using integer chips.



    The product is −36.

    The change in altitude of the aircraft is −36 m in the period of 12 s. That is, the plane will descend 36 m during this period.

17. Represent the change in time by +8.

    Represent the position of the drill bit—the bottom of the well—after each minute as −2.

    Then the final position of the drill bit should be represented by the product (+4) × (−2).

    Find the product of (+4) × (−2) by using integer chips. You should insert 4 groups of 2 blue integer chips. This will result in a total of 8 blue integer chips.

    These blue integer chips represent −16. So, (+8) × (−2) = (−16).

    The final position of the drill bit will be −16 m measured from ground level.

    The well will be 16 m deep after 8 min.

Going Beyond

19. a. The magic sum is 3.

b. The new magic square is this:

−4	−6	4
6	−2	−10
−8	2	0

Yes, this is a magic square.

The magic sum is −6.

c. The magic sum of the new magic square is −6. In order to make the magic sum −12, it seems you have to multiply each integer in the new square by 2:

−8	−12	8
12	−4	−20
−16	4	0

It turns out that the magic sum of this square of numbers is −12 as it is supposed to be.

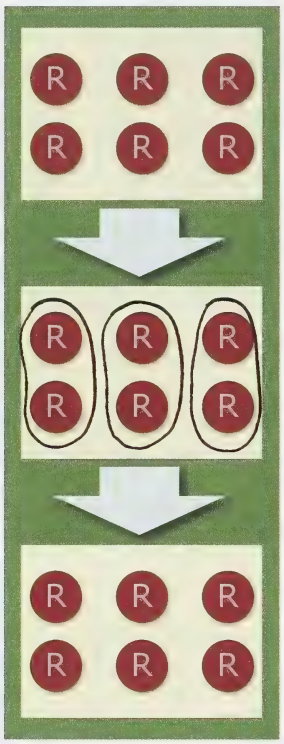
Lesson 2

SC 1.

1.

a. The integer +3 is shown by drawing 3 arrows. The integer +2 is indicated by making each of the arrows 2 units long.

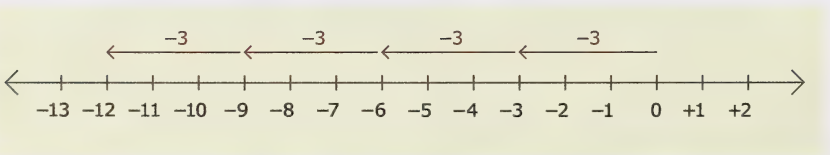
b. Use your integer number chips and record the multiplication process:



The product is +6.

c. The number line shows the product as the point underneath the tip of the third arrow.

2. a.



Starting at 0, 4 arrows are drawn for the first factor +4. Each arrow points to the left and is 3 units long according to the second factor −3.

b. The product is −12. That integer is indicated on the number line by the tip of the arrow drawn last.

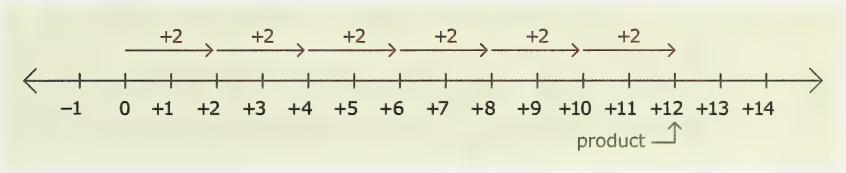
3. Answers will vary. The expressions (−3) × (+2) and (−4) × (−3) have a negative integer (rather than a positive integer) as the first factor. A positive integer as the first factor indicates how many arrows to place on the number line. A negative integer as the first factor suggests the number of arrows to remove. But then you would need something like zero pairs or some other change to the method.

   So, the same method used before to multiply integers using a number line cannot be used here.

SC 2.

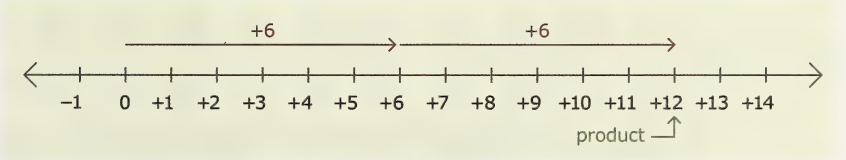
4.

a. **Table row 1:** (+6) × (+2) = ?



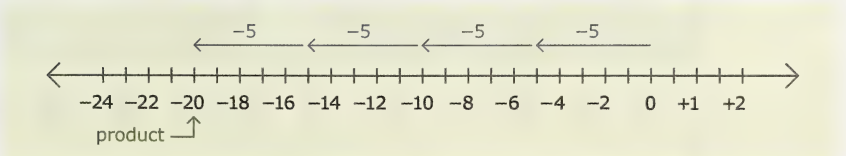
So, (+6) × (+2) = +12.

(+2) × (+6) = ?



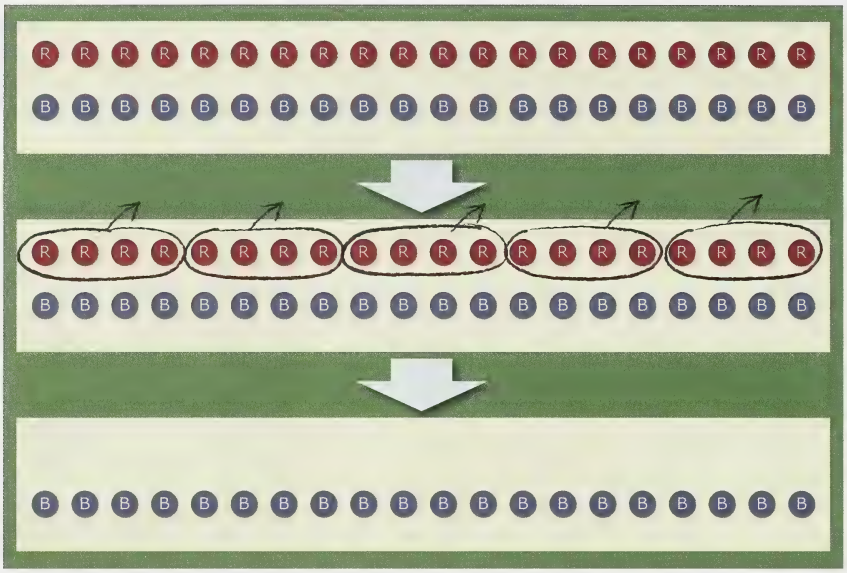
So, (+2) × (+6) = +12.

Table row 2: (+4) × (−5) = ?



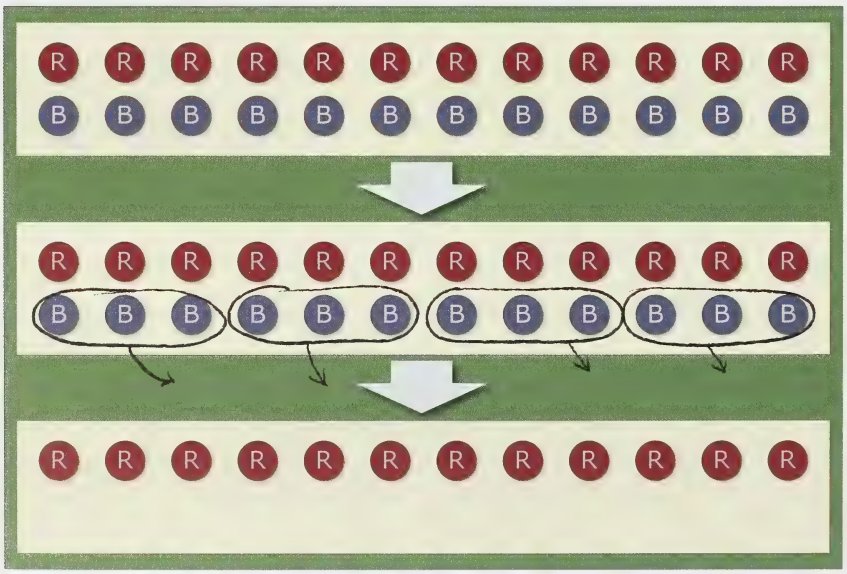
(+4) × (−5) = −20

$(-5) \times (+4) = ?$



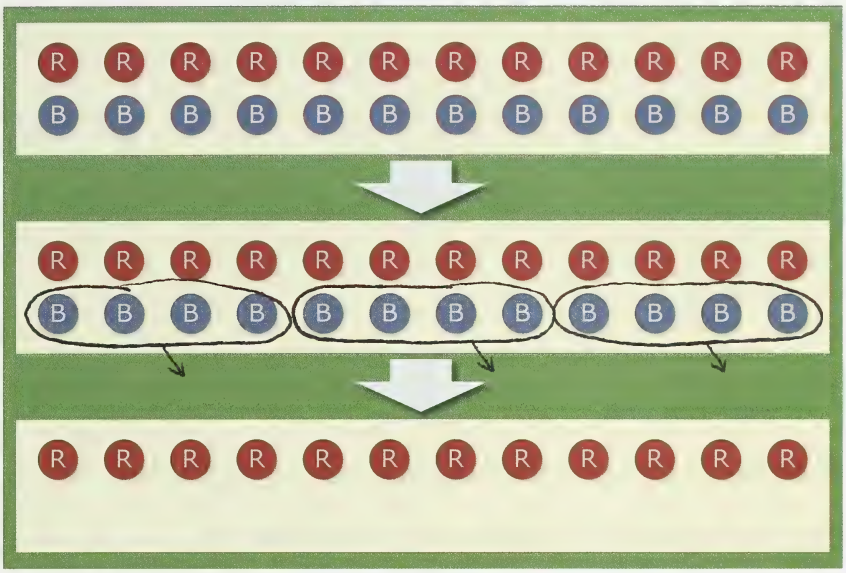
$(-5) \times (+4) = -20$

Table row 3: $(-4) \times (-3) = ?$



So, $(-4) \times (-3) = +12$.

$(-3) \times (-4) = ?$



So, $(-3) \times (-4) = +12$.

b. Each row of the table shows that the left-cell product is equal to the right-cell product. That is evidence that you can multiply factors in either order and still get the same answer. Testing your conclusion with other integer multiplications will support your conclusion.

c. Using a number line to multiply $(-3) \times (+2)$ does not work so well since the first factor is a negative integer. But since the order of the factors does not matter, you can switch them around to $(+2) \times (-3)$. The product of this expression can be found using a number line and will be the same as $(+2) \times (-3)$.

SC 3.

a. $(+4) \times (+7)$

$4 \times 7 = 28$

The factors have the same sign, so the product is positive.

b. $(+3) \times (-10)$

$3 \times 10 = 30$

The factors have opposite signs, so the product is negative.

So $(+3) \times (-10) = -30$.

c. $(-8) \times (-2)$

$8 \times 2 = 16$

The factors have the same sign, so the product is positive.

$(-8) \times (-2) = +16$

d. $(-4) \times (+9)$

$4 \times 9 = 36$

The factors have opposite signs, so the product is negative.

$(-4) \times (+9) = -36$

SC 4. Represent the deduction from Duane's bank account by the integer −65.

Represent the number of monthly deductions by the integer +18.

Then the total of the deductions can be represented by the integer multiplication (+18) × (−65).

$$18 \times 65 \doteq 20 \times 70$$
$$(+18) \times (-65) \doteq -1400 \text{ (approximately)}$$

(+18) × (−65) = −1170 (exactly)

This is quite close to the estimate.

Duane's account balance changed by an amount of −$1170. The negative sign shows the balance decreased.

The total of the deductions from Duane's account during the over the 18-month period is $1170.

SC 5. The factors could be −11 and +7 or +11 and −7.

SC 6.

4.

a. The 2 arrows pointing to the right each 4 units long corresponds to (+2) × (+4).

b. The 3 arrows pointing to the left each 5 units long corresponds to (+3) × (−5).

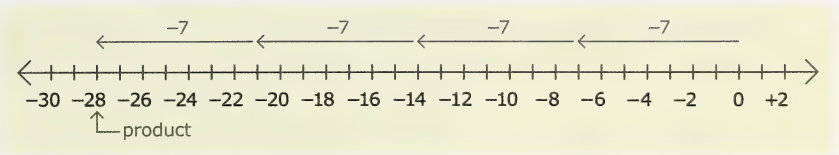
5.

a. The 2 arrows pointing down each 6 units long corresponds to $(+2) \times (-6)$.

b. The 4 arrows pointing up each 4 units long corresponds to $(+4) \times (+4)$.

7.

a. $(+4) \times (-7) = ?$



$(+4) \times (-7) = -28$

9.

a. $(-6) \times (-6) = +36$

d. $(+11) \times 0 = 0$

10. Estimates will vary.

a.

Estimate:	**Integer multiplication:**
$(+17)\times(-24) \doteq (+20)\times(-20)$ $\doteq -800$	$(+17) \times (-24) = -799$

d.

Estimate:	**Integer multiplication:**
$(+28)\times(-47) = (+30)\times(-50)$ $= -1500$	$(+28) \times (-47) = -1316$

13. Represent the change in altitude of the balloon each min by the integer −60.

    Represent the change in time by the integer +25.

    Then the total change in altitude of the balloon is represented by the integer multiplication (+25) × (−60).

    $(+25)\times(-60) \doteq (+20)\times(-60)$
    So, $(+25)\times(-60) \doteq -1200$ (by estimation).
    $(+25)\times(-60) = -1500$ (by integer multiplication)

    The integer multiplication is reasonably close to the estimate.
    The balloon's altitude changes by the integer value−1500 m over the 60-min time interval.

    The negative sign indicates a drop in altitude or a descent. So the balloon's descended by −1500 m.

19.

    a. Yes, and in two ways.

       +4 = (+2) (+2) and

       +4 = (−2) (−2)

    b. The integer −4 cannot be written as a product of two equal integers. First, it's clear neither of two integers having a product of −4 can be equal to zero. Second, according to the sign rule, one of the integers must be negative and the other positive in order for the product to be a negative integer. But two non-zero integers with opposite signs cannot be equal.

       So, you can conclude that −4 cannot be expressed as a product of two equal integers.

23. Answers will vary.

    An artist found that as a rule, her pencil lead shortened by 6 mm each day of work. After 5 days of work, how much will the length of her pencil lead change?

    Check:

    Represent the change in pencil lead each day by the integer −6.

    Represent the change in time—the number of days—by the integer +5.

    So the total change in length of pencil lead is represented by the integer multiplication (+5) × (−6).

Going Beyond

28.

a. The magic product is found by multiplying the digital entries in each of the rows, then in each columns, and then in each diagonal. All yield the same product of –216.

b. The square of numbers becomes this:

−24	+2	−36
+18	+12	+8
−4	+72	−6

The products for rows, columns, and diagonals are +1728, so this is the magic product. This new square of numbers is also a magic multiplication square.

c. The square of numbers after adding −5 is changed to the following:

			D1
+7	−6	+13	R1
−14	−11	−9	R2
−3	−41	−2	R3
C1	C2	C3	D2

The products for the diagonals (D1 and D2), rows (R1 to R3), and columns (C1 to C2) are shown below. These products are no longer equal to each other.

			+429
+7	−6	+13	−546
−14	−11	−9	−1386
−3	−41	−2	−246
+294	−2706	+234	+154

The result is no longer a magic multiplication square. Why? Adding a constant amount has a greater relative affect on integers closer to zero. So the addition affects the magic products in different ways.

Lesson 3

SC 1.

1.

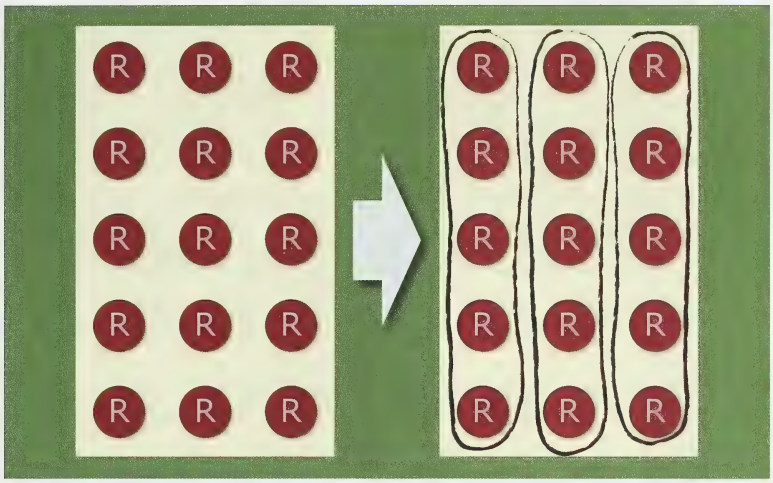
a. The diagram shows how many groups of 2 red integer chips there are in 8 red integer chips.

b. $(+8) \div (+2) = +4$

c. The diagram shows how many integer chips—and their colour—there are in each group when 8 red integer chips are divided into 4 groups.

d. $(+8) \div (+4) = +2$

2. a. Place 15 red integer chips on a surface and separate them into groups of 5. The number of groups will be the quotient.



There are 3 groups.

b. $(+15) \div (+5) = +3$

c. The model shows that when the integer chips are separated into 3 groups, each group contains 5 red integer chips.

This indicates that $(+15) \div (+3) = +5$.

3. a. The diagram shows how 8 blue integer chips can be separated into groups of 2 blue integer chips. In this way, 4 groups are formed. This model indicates the quotient is +4.

b. $(-8) \div (-2) = +4$

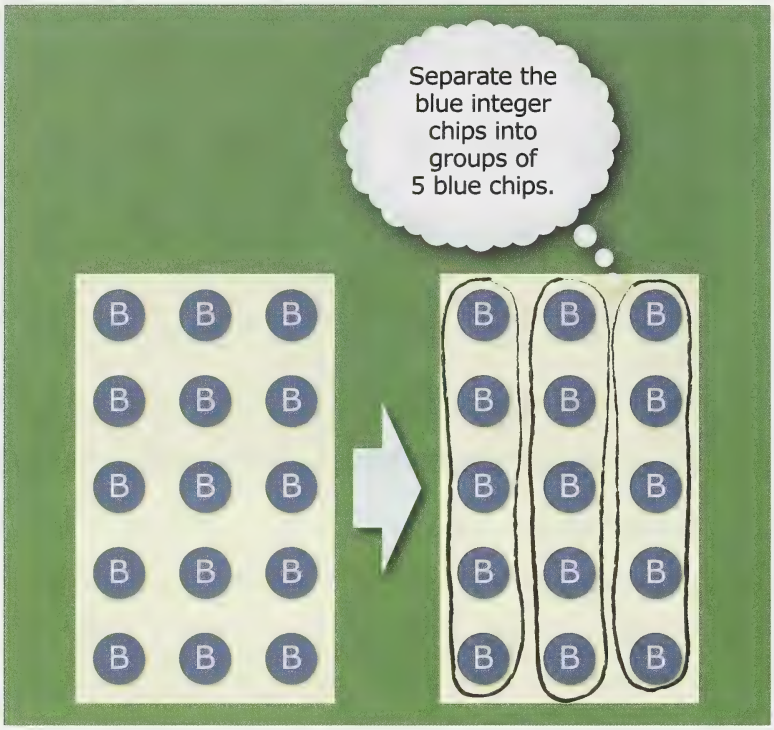
c. The diagram shows 8 blue integer chips separated into 4 groups. Each of the groups made this way contain 2 blue chips. The content of each group corresponds to the quotient of $(-8) \div (+4)$.

d. $(-8) \div (+4) = -2$

SC 2.

4.

a.



There are 3 groups so the quotient is 3.

b. $(-15) \div (-5) = +3$

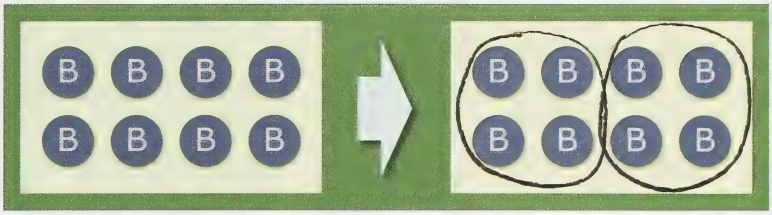
c. The diagram shows that with the integer chips divided into 3 groups, each group has 5 blue integer chips.

$(-15) \div (+3) = -5$

SC 3.

5.

a.



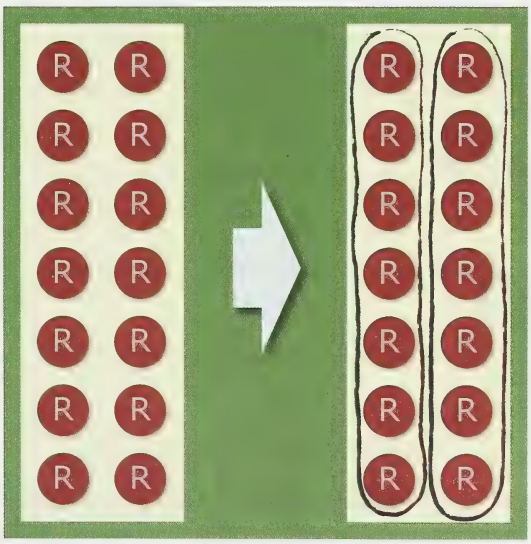
You start out with 8 blue integer chips because the dividend is negative and the numeral part is 8. You split these into 2 groups because the divisor is +2. Then you look at the make-up of the groups. In each group, there are 4 blue tiles.

b. The blue colour of the tiles indicates that the quotient is negative and the number of tiles in each group indicate the numerical part of the quotient is −4.

So (−8) ÷ (+2) = −4.

SC 4.

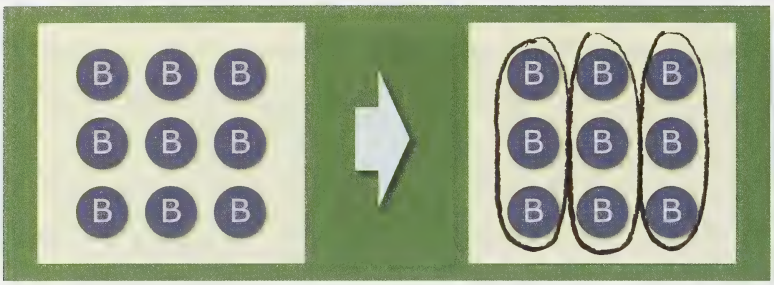
a. (+14) ÷ (+7) = ?



When the 14 red integer chips are separated into groups containing 7 red integer chips, 2 groups are formed. So the quotient is +2.

(+14) ÷ (+7) = +2

b. $(-9) \div (-3) = ?$

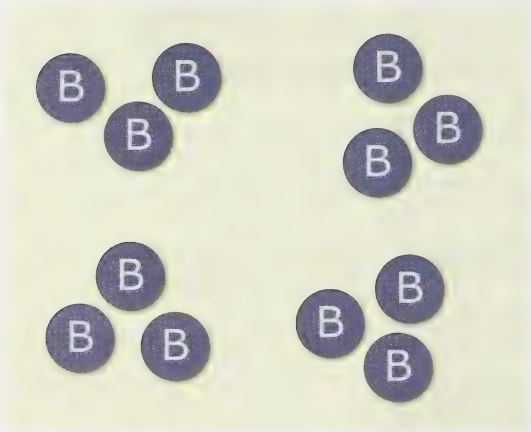


When the 9 blue integer chips are separated into groups each containing 3 blue integer chips, 3 groups are formed. So +3 is the quotient.

$(-9) \div (-3) = +3$

SC 5. The problem can be solved by integer division.

Let the change in temperature each hour be represented by the integer −3. The total temperature change can be represented by the integer −12. The number of hours for this change to happen can be represented by the quotient of $(-12) \div (-3)$. This division can be modelled by integer chips.



When the 12 blue integer chips are separated into groups containing 3 blue integer chips, 4 groups are formed. So the quotient is +4.

$(-12) \div (-3) = +4$

It took 4 h for the temperature to fall 12°C.

SC 6. With red integer chips, the dividends would be positive integers. That's because the groups would consist of red integer chips. The numerical values of the statements remain the same.

So, with red integer chips the integer statements would be these:

$(+8) \div (+2) = +4$
$(+8) \div (+4) = +2$

SC 7.

3.

a. There are 5 groups, so $(+10) \div (+2) = +5$.

b. There are 4 groups, so $(-16) \div (-4) = +4$.

c. There are 7 blue integer chips in each group so $(-14) \div (+2) = -7$.

4.

a. There are 2 groups formed, so $(-4) \div (-2) = +2$.

b. There are 3 red integer chips in each of the groups, so $(+9) \div (+3) = +3$.

According to another way of looking at the model, there are 3 groups formed, so $(+9) \div (+3) = +3$.

c. Each of the groups contains 2 blue integer chips, so $(-12) \div (+6) = -2$.

6.

a. In making groups of 5 red integer chips, you end up with 3 groups, so $(+15) \div (+5) = +3$.

In making 3 equal groups, you end up with 5 red integer chips in each group. So, $(+15) \div (+3) = +5$.

b. In making groups of 9 blue integer chips, you end up with just 2 groups, so $(-18) \div (-9) = +2$.

In making 2 equal groups, you end up with 9 blue integer chips in each group. So $(-18) \div (+2) = -9$.

SC 8.

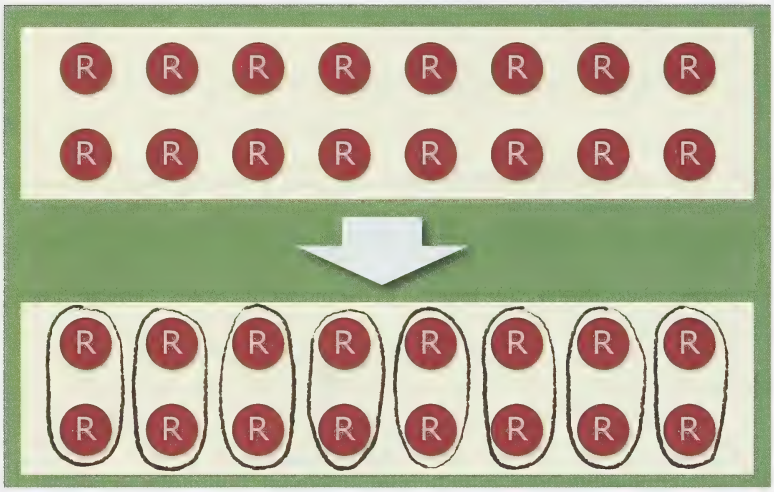
11. Integer division can be used to solve the problem.

    Let the integer +4 represent the number of bus trips each day. Then the number of bus trips taken over the two days of a weekend is represented by the integer +8.

    The total number of dollars spent on a weekend is represented by +16.

    The number of dollars spent for each trip can be represented by the quotient of (+16) ÷ (+8).

    This division can be modelled by integer chips as follows:



    There are 2 red integer chips in each of the groups.

    So, (+16) ÷ (+8) = +2.

    The number of dollars spent for each trip is $2.

12. a. $(-12) \div (-3) = +4$

$(-9) \div (-3) = +3$

$(-6) \div (-3) = +2$

$(-3) \div (-3) = +1$

$(0) \div (-3) = ?$

$(+3) \div (-3) = ?$

$(+6) \div (-3) = ?$

In the list from one division statement to the next, the dividend changes by -3 and the quotient changes by -1.

b. According to the pattern, the final statements should be completed as follows:

$(0) \div (-3) = 0$

$(+3) \div (-3) = -1$

$(+6) \div (-3) = -2$

Going Beyond

16. In order to make the prediction, calculate the temperature change during the 6-h period and then the temperature change in an hour.

To find the temperature change over the 6-h period, subtract the final temperature from the initial temperature for the period.

$(-11) - (+1) = -12$

The temperature drops 12°C during the 6-h period.

Represent this temperature change by the integer -12.

Represent the number of hours by $+6$.

Represent the change in temperature each hour by $(-12) \div (+6)$.

$(-12) \div (+6) = -2$

The change in temperature each hour is -2°C.

Represent the change in temperature during a 3-h period by $(+3) \times (-2)$.

$(+3) \times (-2) = -6$

The change in temperature during the next 3-h period is −6°C.

The temperature at the end of the 6-h period is −11°C.

Represent the temperature at the end of the next 3-h period as $(-11) + (-6)$.

$(-11) + (-6) = -17$

The temperature at the end of the next 3-h period will be −17°C.

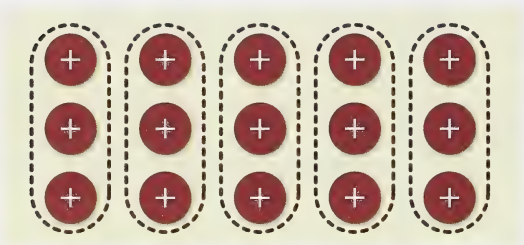
The assumption is that the temperature will continue to drop at the same rate as during the 6-h period.

Lesson 4

SC 1.

1.

a. Answers may vary. Example: The dividend +15 is shown as an arrow 15 units long extending to the right from 0 on the number line. The divisor +3 is shown by sections 3 units long marked on the dividend arrow.

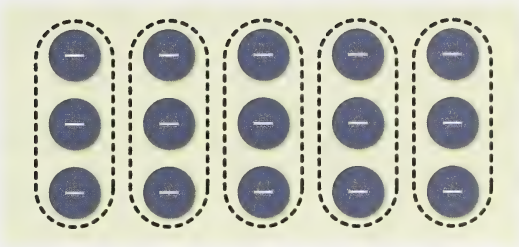
b. 

The quotient is +5.

c. Answers may vary. Example: The quotient is shown by the number of three-unit sections in the dividend arrow.

d. Answers may vary. Example: Since the dividend arrow is divided into 5 sections, the number of units in each section represents the quotient +3.

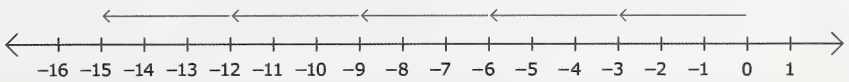
2. a. Answers may vary. Example: The dividend −15 is shown as an arrow 15 units long extending to the left from 0 on the number line. The divisor −3 is shown by sections 3 units long marked on the dividend arrow.

b. 

The quotient is +5.

c. Answers may vary. Example: The quotient is shown by the number of three-unit sections on the dividend arrow.

d. Answers may vary. Example: Since the dividend arrow is divided into five sections, the total value of the units in each section represents the quotient +3.

3. a. 

Explanations may vary. Example: The dividend −15 is shown as an arrow 15 units long extending to the left of 0. The divisor +3 is shown by sections three units long marked on the dividend arrow.

b. $(-15) \div (+3) = -5$

c. Explanations may vary. Example: Since the dividend arrow is divided into five sections, the number of units in each section represents the quotient −3.

4. Explanations may vary. Example: No, a value of +15 cannot be divided using a number line into sections of −3, nor can it be divided into −3 sections.

5.

Multiplication Statement	Related Division Statements	
$(+2) \times (+4) = (+8)$	$(+8) \div (+4) = (+2)$	$(+8) \div (+2) = (+4)$
$(+6) \times (+2) = (+12)$	$(+12) \div (+2) = (+6)$	$(+12) \div (+6) = (+2)$
$(+3) \times (-5) = (-15)$	$(-15) \div (-5) = (+3)$	$(-15) \div (+3) = (-5)$
$(-3) \times (+6) = (-18)$	$(-18) \div (+6) = (-3)$	$(-18) \div (-3) = (+6)$
$(-5) \times (-4) = (+20)$	$(+20) \div (-4) = (-5)$	$(+20) \div (-5) = (-4)$
$(-1) \times (-9) = (+9)$	$(+9) \div (-9) = (-1)$	$(+9) \div (-1) = (-9)$

6. The quotient of two integers with the same sign is positive.

   The quotient of two integers with different signs is negative.

SC 2. Strategies may vary. You can use a number line, the sign rule for division, or a calculator to arrive at the integer value of the quotient for each equation.

a. +3

b. −3

c. +4

d. −6

SC 3. Represent the number of dollars Pierre paid by the integer +42.

Represent the number of persons admitted into the museum as +3.

Represent the cost of each admission by the integer division (+42) ÷ (+3).

(+42) ÷ (+3) = +14

The cost of each admission is $14.

Check:

(+3) × (+14) = +42

SC 4. The red arrow would end at +12, so +12 would be the dividend. Each section of the red arrow would represent +4. So +4 can be thought of as the divisor. There are 3 arrows, which indicate that +3 is the quotient. So, (+12) ÷ (+4) = +3 is one integer statement that is modelled.

As well, you can think of the number of arrows section as indicating a divisor of +3. All the arrow sections—each four units long and coloured red—represent a quotient of +4.

So, (+12) ÷ (+3) = +4 is the other division statement that is modelled.

SC 5.

5.

a. (+18) ÷ (+2) = +9

   (+18) ÷ (+9) = +2

b. (−12) ÷ (−3) = +4

   (−12) ÷ (+4) = −3

6. a. $(-10) \div (-2) = +5$

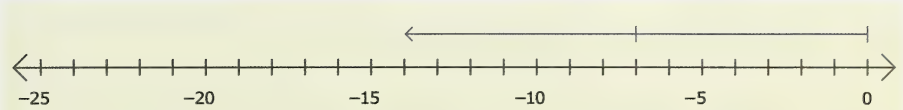
$(-10) \div (+5) = -2$

b. $(+16) \div (+2) = +8$

$(+16) \div (+8) = +$

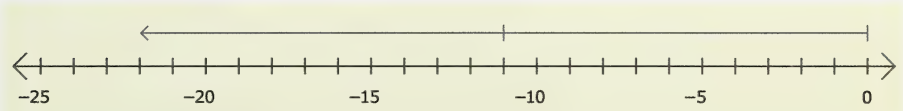
8.

a.



$(-14) \div (-7) = +2$

c.



$(-22) \div (+2) = -11$

9.

a. $(+20) \div (+5) = +4$

Check:

$(+4) \times (+5) = +20$

c. $(-57) \div (+19) = -3$

Check:

$(-3) \times (+19) = -57$

10.

a. $(-26) \div (-26) = +1$

c. $0 \div (-33) = 0$

SC 6.

12.

a. Represent the time duration in minutes by +16.

Represent the distance (down) of dive in metres by the integer −96.

Represent the distance of dive in each minute by the division (−96) ÷ (+16) = −6.

The submarine dove −6 m per minute of the dive.

b. Represent the time duration in minutes by +12.

Represent the distance (up) of the dive in metres by the integer +96.

Represent the distance of dive in each minute by the division (+96) ÷ (+12) = +8.

The submarine travelled +8 m per minute of the climb.

16. Based on the sign rule, only the quotient of (+2408) ÷ (−43) is a negative integer. So the quotient with the least value is (+2408) ÷ (−43).

19. The following is a sample word problem:

An elevator goes down 80 m at a rate of 16 m/s. How long does the elevator take to make this trip?

Check:

Solution would start this way.

Represent the distance the elevator moves by the integer −80.

Represent the distance of travel in each second by the integer +16.

Then you would represent the duration of the trip in seconds by the integer division (−80) ÷ (+16) = −5.

Going Beyond

22. Guess and Check Table Values

Larger Integer	Smaller Integer	Quotient	Sum
+18	−6	(+18) ÷ (−6) = −3	+12
+21	−7	(+21) ÷ (−7) = −3	+14
+24	−8	(+24) ÷ (−8) = −3	+16
+27	−9	(+27) ÷ (−9) = −3	+18
+30	−10	(+30) ÷ (−10) = −3	+20
+33	−11	xx	xx

So, the two integers are +30 and −10.

Lesson 5

SC 1.

1. Explanations may vary. Example: 6 × (−15) represents the dive for the first 6 min when the rate was 15 m/min. Similarily, 20 × (−25) represents the dive for the next 20 min when the rate was 25 m/min.

2. Abeni evaluated the expression correctly. Explanations may vary. Example: The multiplication is performed before the addition.

3. Explanations may vary. Example: Laura evaluated the expression by performing the operations from left to right. Rob added −15 to +20 before carrying out the multiplication.

4. The depth of the submersible after the dive is 590 m.

$$\begin{aligned} &6\times(-15)+20\times(-25) && \text{multiply in order, left to right}\\ &=(-90)+20\times(-25) && \text{multiply before addition}\\ &=(-90)+(-500) && \text{add}\\ &=-590 \end{aligned}$$

Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronoto: McGraw-Hill Ryerson 2008), 425. Reproduced by permission.

SC 2. Strategies may vary. You can use a number line, the sign rule, or a calculator to calculate the operations in the expressions.

a. 20
b. 2
c. −19

SC 3. Place the integer values of the temperatures in the numerator of the fraction. Place the integer representing the number of days in the denominator. Then translate the fraction into a division expression (as in "Example 2") and evaluate the expression using the proper order of operations.

$$\frac{(-7)+0+1+(-6)}{4}$$

$$[(-7)+0+1+(-6)] \div 4$$

Brackets are used keep the integers in the numerator grouped together. The brackets are made square since they surround integers having round brackets.

$= (-12) \div 4$

Once the operations within the square brackets are done, the square brackets can be left off. Only the round brackets used to surround the integer −12 remain.

$= -3$

Your final answer should be expressed as follows:

The mean daily low temperature for the 4 days was −3°C.

SC 4. Brackets are used to group integers, so the operation of grouped integers is done earlier. If the grouped integers have brackets, then square brackets, not round brackets, are used to do the grouping. The square brackets make the integer expression easier to read. For example, (−2) x [(3 + 5) ÷ 2] is easier to read than (−2) × ((3 + 5) ÷ 2).

SC 5. Only final answers are shown.

4.
   a. +17
   b. −11

5.
   a. −3
   b. 0

6.
   b. 14
   c. 10

7.
   c. +3

SC 6.

8. The mean temperature is the sum of the temperatures divided by the number of temperature readings.

$$\frac{(-4)+(+1)+(-2)+(+1)+(-6)}{5}$$

$= [(-4) + (+1) + (-2) + (+1) + (-6)] \div 5$

$= (-10) \div 5$

$= -2$

The mean daily low is −2°C.

11. The mean of 5 integers is found by dividing the sum by 5.

So the sum of the integers divided by 5 is −11.

So the sum must be 5 times the mean.

Represent the sum as $5 \times (-11)$.

This expression equals −55.

The sum of the integers is −55.

16.

a. Represent the rate of descent during the first 3 min by the integer −50.
Represent the change in altitude during this time as $3 \times (-50)$.

Represent the rate of ascent during the next 2 min by the integer +100.

Represent the change in altitude during this time as $2 \times (+100)$.

Represent the overall change for the 5 minutes as $3 \times (-50) + 2 \times (+100)$.

$$\begin{aligned} 3\times(-50)+2\times(+100) &= -150+200 \\ &= +50 \end{aligned}$$

The overall change over the 5-min period was 50 m up.

17. The period of time from 8:00 p.m. to 10:00 a.m. the next day is 14 h.

Represent the length of this period by the integer +14.

Represent the time (in minutes) lost each hour by the watch as −9.

Represent the time lost by the watch in the 14 h time period as $14 \times (-9)$.

$14 \times (-9) = 126$

So the watch loses 126 minutes during the time interval.

That loss equals 2 h 6 minutes.

The reading of the watch at 10:00 a.m. the next day without a loss would be 10:00.

Subtract 2h 6 min from this reading.

Represent 10:00 a.m. as 9 h 60 min.

So the reading of the watch will be $(+9 \text{ h} + 60 \text{ min}) - (+2 \text{ h} - 6 \text{ min}) = +7 \text{ h} + 54 \text{ min}$

The watch will read 7:54 a.m.

20.

a. $20 - 3 \times (-8) = 44$

b. $4 \times 5 + (-2) \times (-3) = 26$

c. $-62 \div (-11 + 9) = 31$

d. $[-3 + (-5)] \times 3 \div (-4) - 13 = -7$

Going Beyond

21. a. $2 \times 3 - 4 \times 5 = -14$

b. $3 \times [14 + (-2)] - 30 = 6$

Unit 2 Summary

SC 1.

1.

a. Answers may vary. Example: The three blue chips represent −3°C and the eight red chips represent 8°C.

b. 5°C. Explanations may vary. Example: Pairing each blue chip with a red chip leaves five red chips, which represent 5°C.

2. a. Answers may vary. Example: The red arrow indicates the starting temperature of 4°C, and the blue arrow indicats a decrease of 9°C.

b. −5°C. Explanations may vary. Example: Counting down nine units on the number line from +4 finishes at −5°C.

c. Answers may vary. Example: The diagram could use four red chips to represent the starting temperature and nine blue chips to represent the decrease of 9°C. Pairing each chip with a blue chip leaves five blue chips. These chips represent −5°C.

3. Answers may vary. Example: The total temperature change is −20°C. Represent this change with 20 blue chips. Divide the 20 chips into four groups. Each group will contain five blue chips. Therefore, the temperature change would be −5°C/h.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008) 390. Reproduced by permission.

SC 3. Turn to pages 500 and 501 of the Answers section of your textbook. Check your answers with those provided.

SC 4.

1. C

   $(-5) + (-5) + (-5) + (-5)$ is equivalent to $(+4) \times (-5)$

2. C

   The diagram shows 12 zero pairs with 6 groups of 2 red chips per group being removed. There are 12 blue chips remaining. This represents $(+6) \times (-2) = -12$.

3. B

   To determine which expression does not equal 3, evaluate each expression.

   $A: (-3) \times (-1) \div (+1)$ Multiply and divide in order, left to right.
   $= 3 \div (+1)$
   $= 3$
   $B: (+3) \div (-1) = -3$
   $C: (+27) \div (+9) = 3$
   $D: (+27) \div (-3) \div (-3) = 3$

4. D

   To determine which expression equals $(-3) \times (+8) = -24$, evaluate each expression.

   A: $(-12) \times (-2) = 24$
   B: $(-24) \div (-1) = 24$
   C: $(+4) \times (+6) = 24$
   D: $(+72) \div (-3) = -24$

5. C

   To determine the greatest product of any two integers, select two integers with the same sign to obtain a positive product.

   $(-22) \times (-16) = 352$
   $(+18) \times (19) = 342$

   Any other pairs will produce smaller products. So, the greatest product is 352.

6. A

   To determine the value of the expression, follow the rules of the order of operations.

   $2 \times [5 - (-3)] + (-6)$ Brackets.
   $= 2 \times 8 + (-6)$ Multiply.
   $= 16 + (-6)$ Add.
   $= 10$

7. Dividing any integer by its opposite results in a quotient of −1. Example: 8 and −8 are opposites.

   $8 \div (-8) = -1$

8. To determine the overnight low, determine how much the temperature dropped in the 6-hour period from midnight until 6:00 a.m. To do this, multiply −2 by 6.

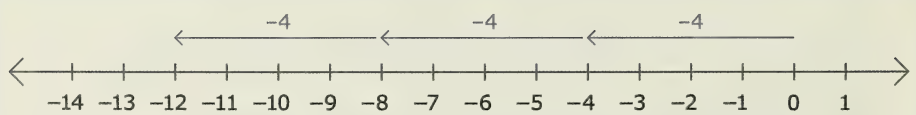
   $-2 \times 6 = -12$

   The temperature changed by −12°C in the 6-hour period.

   Next, add this change to the temperature at midnight of +3°C.

   $-12 + 3 = -9$

   The overnight low temperature was −9°C.

9. 

   Answers may vary. Example:

The number line shows −12 divided into 3 equal parts.

$(-12) \div (+3) = -4$

The number line shows −12 divided into 3 equal parts, each representing −4.

$(-12) \div (-4) = +3$

10. a. The integers −65 and +18 have different signs, so the product is negative.

$-65 \times 18 = -1170$

b. The integers −24 and −31 have the same sign, so the product is positive.

$-24 \times (-31) = 744$

11. a. The integers −64 and −16 have the same sign, so the quotient is positive.

$-64 \div (-16) = 4$

b. The integers +99 and −11 have different signs, so the quotient is negative.

$99 \div (-11) = -9$

12. a.
$$\begin{aligned} & -6 \times 5 + (-27) \div (-9) && \text{Multiply and divide in order, from left to right.} \\ &= -30 + (-27) \div (-9) \\ &= -30 + 3 && \text{Add.} \\ &= -27 \end{aligned}$$

b.
$$\begin{aligned} & [8 + (-6)] \div (-2) - 4 \times (-3) && \text{Brackets.} \\ &= 2 \div (-2) - 4 \times (-3) && \text{Divide and multiply in order, from left to right.} \\ &= -1 - 4 \times (-3) \\ &= -1 - (-12) && \text{Subtract.} \\ &= 11 \end{aligned}$$

13. To determine the mean daily temperature, add the temperatures for the week and divide by 7.

$$\begin{aligned} & [(-6) + 3 + 1 + (-1) + (-3) + (-2) + (-6)] \div 7 && \text{Brackets.} \\ &= -14 \div 7 && \text{Divide.} \\ &= -2 \end{aligned}$$

The mean daily temperature was −2°C.

14. Word problems will vary. Example: Faye borrowed \$8/week from her brother. How much did she owe her brother at the end of 4 weeks?

    To determine how much Faye owes, multiply the number of weeks by how much she borrowed each week.

    $(+4) \times (-8) = -32$

    Faye will owe her brother \$32.

15. Answers may vary. Example: Examine the signs of the two integers.

    The product of two integers with the same sign is positive. Example:

    $-8 \times (-9) = +72$.

    The product of two integers with different signs is negative. Example: $-14 \times (+5) = -70$. The product of two integers will be zero if one or both of the integers are zero.

    Example: $0 \times (-19) = 0$

16. To determine the depth of the submarine after the dive, find the amount it dove at the rate of 12 m/min, by multiplying 12 by 6.

    $12 \times 6 = 72$

    The submarine dove 72 m at the rate of 12 m/min.

    Find the amount it dove at the rate of 7 m/min, by multiplying 7 by 4.

    $7 \times 4 = 28$

    The submarine dove 28 m at the rate of 7 m/min.

    The total amount of the dive was $72 + 28 = 100$ m.

17. a. To determine the account balance at the end of the two months, compute the following:

$$\begin{aligned}
&200+4\times 95+8\times(-50)+2\times(-10) && \text{Multiply and divide in order, from left to right.}\\
&=200+380+8\times(-50)+2\times(-10)\\
&=200+380+(-400)+2\times(-10)\\
&=200+380+(-400)+(-20) && \text{Add in order, from left to right.}\\
&=580+(-400)+(-20)\\
&=180+(-20)\\
&=160
\end{aligned}$$

The account balance at the end of the two months is $160.

b. To determine when Peter's account will be empty if he continues in the same way, compute the change in his account per month.

$$\begin{aligned}
&\left[4\times 95+8\times(-50)+2\times(-10)\right]\div 2 && \text{Brackets; multiply within the bracket}\\
&=\left[380+8\times(-50)+2\times(-10)\right]\div 2 && \text{in order, from left to right.}\\
&=\left[380+(-400)+(-20)\right]\div 2\\
&=\left[380+(-400)+(-20)\right]\div 2\\
&=-40\div 2 && \text{Divide.}\\
&=-20
\end{aligned}$$

The amount of change in Peter's account per month is −$20. Divide the amount that is in Peter's account $160 by the amount of change in his account per month.

$160 \div -20 = -8$

If Peter continues in the same way, his account will be empty in 8 months.

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